Georgia State University

Department of Computer Science

CSc 4330/6330 Kebina Manandhar

Fall 2018 **Assignment 3 A, Answers**

(69 points total)

1. (16 points) Make all parentheses explicit in the following expressions:

a. λx. ( (xz) (λy.(xy)) )

b. (λx.xz) ( λy.(w (λw.(((wy)z)x) )) )

c. (λx. (x (x y) (λx.(yx)) ) )

1. For each of the following terms, identify the free variables in each term and identify which terms are closed:
2. FV(M)={x},

No terms are closed

1. FV(M)={y}-{y}

No free variable,

Y is closed

1. FV(M)={x,y}-{x}

=ans: {y}

No terms are closed

1. FV(M) =FV(y,x) – {x}

={y,x}U{x}

=ans: {x,y}

No terms are closed

1. FV(M)=FV(w λw.wyzx) –{y}

=FV(w) U FV(λw.wyzx) – {y}

={w} U FV (wyzx) – {w} – {y}

={w} U{w,y,z,x} – {w} – {y}

={w} U {z,x}

= {x,y,w}

No terms are closed

1. FV(M)=FV(xz λy.xy)-{x}

={x,z}U{x,y}-{y}-{x}

={x,z,y}-{y}-{x}

={z}

No terms are closed

1. ((((λx.x)z)x)(( λy.(z y))y))

FV(M)=FV(x) U FV(z)U FV(x) U (( λy.(z y))y)) –{x}

={x,z,x} U FV(z) U FV(y) U FV(y) –{y} –{x}

={x,z,x} U FV(z,y,y) –{y} –{x}

={x,z,x} U {z,y,y} –{y} –{x}

={x,z,x,y,y}-{y}-{x}

= ans: {x,z,y}

No terms are closed

1. Using the terms from above, apply the following substitutions and show the resulting expression:
2. = (λy.y)
3. = (λy.y) ,no change
4. = ((λx.y) (λx.(x y)))
5. λx.xz λy.xy ,no change
6. = ((((λx.x)(λy.y))x)(( λy’.( (λy.y)y’))y))

4. (19 points )Reduce the following expressions to values:

a. (3 points)

(((λx. (λy (\*x y)) 5) 6)

=> ( (λy (\*5 y)) 6)

=> (\* 5 6)

= 30

b. (3 points)

(λx. x+1) ((λy. y + 2) 3)

=> (λx. x+1) (5)

=> (5+1)

=6

c. (7 points)

(λfab(f a b))(λxy (+ x y))((λw. (\*w w))5)((λv. (\*vv))2)

=> (λfab(f a b))(λxy (+x y))((λw. (\*w w))5) (4)

=> (λfab(f a b))(λxy (+x y))(25)(4)

=> (λab( (λxy (+x y)) a b))(25)(4)

=> (λb ( (λxy (+x y)) (25) b))(4)

=> ( (λxy (+x y)) (25) (4))

( (λy(+25 y))(4))

=> (+25 4)

=29

d. (6 points)

((( (λf (λx ((f x) f))) (λy (λg (g (+ y y)))))2) (λa a))

=> (((( λfx ((f x) f)) (λyg (g (+ y y))))2)(λa a))

=> (( λx ( ((λyg (g (+ y y)))x) (λyg (g (+y y))))) 2) (λa a)

=> (( (λyg (g (+y y)))2)(λyg (g (+ y y))))(λa a)

=> (( λg(g (4))) (λyg (g (+y y)))) (λa a)

=> (( λyg (g (+y y)))(4))(λa a)

=> (λg (g (8)))(λa a)

=>((λa a)(8))

= 8

5. (5 points)

((((λx.x)z)x)(( λy.(z y))y))[z := (λx.(x y))]

= ((((λx.**x**) (λx.(x y)))x)(( λy’.( (λx.(x y))y’))y))

= (((λx.(**x** y))x)(( λy’.( (λx.(x y))y’))y))

= (x y)(( λy’.( (λx.(**x** y))y’))y)

= (x y)(( λy’.(**y’ y**))y)

= (x)(y y)